International Journal of Engineering, Science and Mathematics
Vol. 10 Issue 9, September 2021,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

SYMMETRIES AND CONSERVATION LAWS IN ELEMENTARY PARTICLE

LALIT KUMAR

Department of Physics, Meerut College Meerut, Chaudhary Charan Singh University Meerut, Uttar Pradesh- 250001, India.

KEYWORDS: Symmetry, Conservation laws, U(1), SU(2) and SU(3) etc.

,

ABSTRACT

Role of symmetry is very crucial to explain the various properties and behaviour of the elementary particles. Every symmetry are always associated with corresponding conservation law and it can be either discrete or continuous in nature. The symmetries of the standard model associated with the electromagnetic interaction, weak and strong force are described by the groups U(1), SU(2) and SU(3).Fundamental aspect for some of the symmetry like as permutation symmetry, continuous space-time symmetry, discrete symmetry are elaborated here. higher level of symmetry flavour symmetry, super symmetry and SU(5) symmetry for unification is also discuss here. Various cause of symmetry breaking is discussed here. Consequences of the existence of symmetry is that a corresponding conservation law like as conservation of charge, linear momentum, angular momentum, energy, baryon number, lepton number, parity etc. In this research paper elaboration of various conservation law and its consequences is also done in modern context.

INTRODUCTION:

The Greeks and others have strongly believe that symmetry of objects exist in nature German astronomer Johannes Kepler also had great interest in symmetry and he wants to impose symmetry from the motion of the planet. Isaac Newton UK physicist applied the symmetrical principle like as principle of equivalence of inertial frame Noether's theorem states that every symmetry is corresponding to conservation law. Momentum and energy conservation law have the fundamental importance in many process and such laws are consequences of the dynamic law of nature. James Clerk Maxwell equation which was proposed in 1865 possesses both Lorentz invariance and Gauge invariance. These equations have significant role but these equations were not understood well during four decades. In 1905 Albert Einstein forward the idea about the constraints the dynamical laws, Einstein recognise the symmetry utility in Maxwells equations and he connected it to space time. In 1920 symmetry principle was applied in quantum mechanics successfully. Now a days it is using as a guiding principle for further unification and progress.

Steven WeinbergAmerican theoretical physicist said that it is increasing clear that symmetry group of nature is the deepest thing that we can understand about nature today.

According to Richard Feynman American physicist anything is symmetrical if one can subject it to a certain operation and it appears exactly the same after the operation. Symmetry have become the extremely important consideration of formulation of the physical laws and consequence of the existence of symmetry is that a corresponding conservation law.

Great physicist Tsung Dao Lee said about the symmetry, Since the beginning of physics, symmetry considerations have provided us with an extremely powerful and useful tool in our effort to understand nature. Gradually they have become the backbone of our theoretical formulation of physical laws".

Hermann Weyl German physicist and philosopher have famous quotes about the symmetry, " I am at two things; on the one hand to clarify step by step, the philosophical mathematical significance of the idea of symmetry and on the other, to display the great variety of applications of symmetry in the arts, in inorganic and organic nature".

Types of Symmetry:

There are the four group of symmetries which are highly useful and found importance in physics.

1. **Permutation Symmetry:** This type of symmetry is based upon the statistics and quantum mechanically particles are govern by two statistics

a) **Bose Einstein Statistics**: this type of statistics is applicable to those particles which have spin in integral form- photon (spin 1), Graviton(spin 2) and Helium nucleus (spin 0).

b) **Fermi Dirac Statistics:** this type of statistics is applicable to those particles which have spin in half integral form- proton, electron and Neutron all have half spin.

2. **Continuous Space Time Symmetry:** this type of symmetry is found in translation and rotation etc. and translational invariance leads to law of conservation of linear momentum however rotational invariance leads to law of conservation of angular momentum.

3. **Discrete Symmetry:** symmetries like as space inversion is said to be discrete symmetries. In this symmetry transformation change the sign of spatial coordinate of events and change of coordinates can be interpretated as either an active or a passive transformation. Symmetry like as time reversal, particle antiparticle etc are also an example of this type of symmetry.

4. **Unitary Symmetries:**

a) **U1 Symmetries:** this type of symmetries is linked to conservation of baryon number, lepton number, electronic lepton number μ lepton number, τ lepton number and charge etc.

b) **SU2:** this type of symmetry is highly useful for elementary particle and it is connected to isospin of the elementary particle.

c) **SU3:** this type of symmetry is highly useful for elementary particle and it is connected to color of the elementary particle.

d) **SU(n):**this type of symmetry is highly useful for elementary particle and it is connected to flavor symmetric of the elementary particle.

U1 and SU3 symmetry are exact and all others symmetries like SU2 and SU(n) are broken in various cases.

Fundamental Aspects for some of the symmetry principle

Non Observable	Symmetry Transformation	Conservation Rule
Difference between identical particles	permutations	Bose Einstein and Fermi Dirac Statistics
Absolute Spatial position	Space translation	Linear momentum
Absolute Spatial direction	Rotation	Angular Momentum
Absolute Time	Time translation	Energy
Absolute right or left	Space inversion	Parity
Relative phase between a state of different charge Q	$\Psi \rightarrow e^{iQ\theta}\Psi$	Charge
Relative phase between a state of different Baryon number B	$\Psi \rightarrow e^{iB\theta}\Psi$	Baryon number
Relative phase between a state of different lepton number L	$\Psi \rightarrow e^{iL\theta}\Psi$	Lepton number
Relative phase between a state of different electron lepton number	$\Psi \rightarrow e^{iL_e\theta}\Psi$	e- Lepton number
Relative phase between a state of different μ lepton number	$\Psi \rightarrow e^{iL_\mu\theta}\Psi$	μ- Lepton number
Relative phase between a state of different τ lepton number	$\Psi \rightarrow e^{iL_\tau\theta}\Psi$	τ- Lepton number
Absolute sign of electric charge	$e \rightarrow -e$	Charge conjugation
Difference between different coherent mixture of p-n state	$\binom{p}{n} = u\binom{p}{n}$	Isospin

SU(2) Symmetry:

Proton and neutron behaved as the same way as far as nuclear force is concern but there behaviour is different for electromagnetic interaction because neutron is chargeless and proton have the positive charge. Now we can imagine a group of symmetry operations which could transform a proton into a neutron and vice-versa in the absence of an electromagnetic field. Heisenberg in 1934 suggested the existence of different kind of the spin called isospin. Now for the proton and neutron we can assumed to orthogonal state of the same particle known to be nucleons. Basically isospin state of any particle is described by two quantum numbers I(isospin) and I_3(third component of isospin). For a particular elementary particle 2I+1 represents the number of particles in that particular family. Value of third component of isospin lies between –I to +I separated by unity. In the strong interaction Isospin always remains to be conserved. For the nuclear family there are two members proton and neutrons so 2I+1=2 or I= ½ then proton have the isospin ½ and third component isospin ½ similarly neutron have the isospin ½ and third component isospin -1/2 .

For the proton and neutron isospin states of the isospin symmetry can be described as

$P = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, n= $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

In strong interaction conservation of isospin always occurs due to isospin symmetry however isospin symmetry does not remains conserve in electromagnetic and weak interaction however third component of isospin always conserve in electromagnetic interaction. Breaking of isospin symmetry basically makes a difference between the proton and the neutron. As we know that due to exchange of pi meson strong force is produced and there are three type of pi meson exist in the nucleus and for the three different pions in the isospin state we can described their state as

$$\pi^+ = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \pi^0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \pi^- = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

SU(3) Symmetry:

It is the group of 3× 3 unitary matrix with unit determinant and fundamental multiplet in three dimensional in nature. Octet rule is found in mason and baryon and it is an example of SU(3) symmetry, this type of symmetry are able to explain regularities observed in the hadron world. SU(3) symmetry scheme is highly useful and it is responsible for ω particle which is a hyperon and this particle was predicted by Gell -Mann.SU(3) symmetry is not exact and it is broken in the case of strong interaction.

Various other symmetry invariance and corresponding conservation law

Class	**Invariance**	**Conserve Quantity**
	SU(1) Gauge transformation	Electric charge
	SU(1) Gauge transformation	Lepton
	SU(1) Gauge transformation	Hypercharge
	$SU(1)_{\gamma}$ Gauge transformation	Weak hypercharge
Internal Symmetry	SU(1)× SU(2) Gauge transformation	Electroweak force
	SU(2)× SU(2) Gauge transformation	Isospin
	SU(1)× SU(2) Gauge transformation	Weak transformation
	Partiy×SU(2)	G-Parity
	SU(3)	Baryon number
	Winding number SU(3)	Quark color
	Gauge transformation SU(3)	Quark flavor
	SU(2)× SU(3)	Standard model
Discrete Symmetry	C Charge Conjugation	Charge parity
	P Coordinate Inversion	Spatial parity
	T Time Reversal	Time parity
	CPT	Product of all
Lorentz Symmetry	Translator invariance in space	Linear momentum
	Isotropy in space	Angular momentum
	Homogeneity of time	Energy conservation

Higher Symmetries:

For hadronic behaviour higher symmetries such as SU(4), SU(5) and SU(6) are also taken in to the consideration and when SU(3) combine with SU(6) symmetry and also SU(2) spin symmetry mix together for proton and neutron then magnetic moment ratio for proton to neutron is experimentally observed -1.46 and theoretical value is found to be -1.5 for the same case. SU(4) and SU(5) symmetries are also used to describe spectra of hadron which carrying the charm and bottom quantum number.

SU(5) Symmetry for Unification:

As we know that gravitational interaction is very weak in nature and so unification of gravitational interaction with other interactions are still unsolved problem however we are trying unification of all four interactions with the help of sting theory. Grand unification theories (GUT) unifies all three interactions strong, EM and weak interaction at high energy coupling constant of all interactions approach towards to each other.

Flavour Symmetries:

Isospin symmetry is a flavour symmetry of the strong interaction between up and down quarks. When up and down quarks taken into consideration together then it is described by SU(2) symmetry, and when addition of strange quark takes place then it extended into SU(3) symmetry. Strange quark mass is responsible for partially broken of this type of symmetry and this can be extended up to the SU(6) symmetry between all the quark flavours but level of symmetry breaking also becomes very large in this type of symmetry but cause of breaking of quark flavour symmetry is still unknown

Supersymmetry:

Each boson is teamed up with a fermion as a super partner and vice versa, since now no particle are super partner to each other, one can expect new particle with high mass. Super partner appearance can be expected at LSC (large hadron collider) particularly in neutrino.Basically conversions of coupling constants with super symmetry can be occurs in TeV energy range, super symmetry predict the expectations of super symmetry particle and at such high energy range coupling constant converse in to $1/8\pi$.

Discrete Symmetry Breaking:

In weak interactions, symmetry of C,P and T breaks. Our universe is made up of matter but charge reversal symmetry C requires equal amount of matter and anti matter so charge conjugation does not remains conserve, standard model contains and predicted only left handed neutrino and it is responsible for broken the inversion symmetry P however according to CPT theorem multiplication of C, P ant T remains preserved in each type of interaction.

Minimal Symmetry Breaking:

According to Peter Higgs, in weak interaction SU(2) symmetry breaks because the law if physics are symmetric but our universe is not symmetric in nature.

Symmetry Breaking by the Higg's Field:

Like a magnet, Higg's Field does not favour in any particular direction, non zero average of the Higg's field is responsible for symmetry breaking.

Nobel prize in physics in 2008 has been awarded to YoichiroNambu for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics and M Kobeyashi and T Maskawa for the discovery of the origin of the broken symmetry which predict the existence of at least three family of quark in nature.

Symmetry Breaking in Nuclear Physics:

Symmetry itself is a secret of nature, but many times in nature symmetry breaking takes place in a system. There are many reasons due to which symmetry of nature is hidden and broken, one of reason is dynamics is only approximately symmetric. In nuclear physics isotopic symmetry exist which itself is an example of approximate symmetry. Weakness of electromagnetic force and lesser value of up and down quarks is responsible for approximate symmetry in nuclear force. In nature spontaneous symmetry is also exist which is the hidden symmetry because state of system is not symmetric but law of physics are symmetric in nature.

Symmetry break down confirmed by two team at large hadron collider at CERN near Geneva. Team of scientist had discovered a Higgs like boson and confirmed the symmetry broken mechanism which was suggested by Peter Higgs in 1964 and he had suggested the cause of symmetry broken is Higgs boson. After the confirmation of Peter Higgs theory our standing about symmetry becomes very clear and now it also highly crucial concept in physics.

Electrons and neutrons can be interchanged without changing the law of nature and ultimately it develops the electroweak theory which was proposed by S. Weinberg, S Glashow and Abdus Salam in 1960. This theory unified the electromagnetic interaction with the weak interaction and decay of neutron in to proton in the nucleus take place due to this mechanism. Basically Higgs boson has very significant role in breaking the symmetry of weak interaction.

Protons and neutrons are made up of two type of quarks up quarks and down quarks, nuclear force are strongest force in nature binds protons and neutrons and stabilize the nucleus is also have large tendency to symmetry but law of physics are blind in many cases like as when exchange of up quarks and down quarks and vice-versa take place. Standard model altogether electroweak theory and quantum chromodynamics theory of strong interaction act with in the nucleus, this model unified all three forces weak, electromagnetic and nuclear force altogether.

Internal symmetries are independent upon the position of the particle. Quarks, neutron, electron and proton may be exchanged at different point in spatial and time coordinates without any deep consequences and quanta of corresponding interaction exchanged between different type of interaction like as in electromagnetic interaction quanta isphoton, in strong interaction quanta is pi- meson and in weak interaction quanta is intermediate boson. In electroweak quanta theory photon is joined by W^+, W^- and Z^0 particles, for production of weak force. Gluon exchange takes place between different quarks and this is responsible for nuclear force. Symmetry requirements shows that particle and its interaction are always needed for explanation of any given theory.

Matter- Antimatter Asymmetry of the Universe:

According to Big Bang model of the Universe time goes only in forward direction and never in backward direction. Scientist believed that at the time of Big Bang matter and antimatter were originally created in equal amount. In 1966 Sakharov postulated reason of dominancy of matter in the universe and these reasons are-

1. No thermal equilibrium
2. Baryon number violation
3. CP violation

Conservation Law in Elementary Particle:

1. **Conservation of Charge:** total charge is always conserved in every type of interaction and reactions an there is no exception of this role. For example

$$ {}^{10}_{5}B + {}^{4}_{2}He \rightarrow {}^{13}_{6}C + {}^{1}_{0}n $$

This nuclear reaction shows that initially there are seven protons and same number of protons remains after the product of reaction so we can say that $\sum Z$ always remains to be constant in every nuclear reaction.

Basically conservation of electric charge is corresponding to Gauge transformations which are shifted in the zeroes of the scalar electromagnetic potential V and vector electromagnetic potential A and electric field E is linked to electromagnetic potential V by the expression E= -grad V and magnetic field B is linked to the EM potential A by the expression B= Curl A . basically Gauge transformation leaves electric field E and Magnetic Field B unaffected and this invariance is corresponding to law of conservation of charge. When the scalar potential V and vector potential A are shifted by the gradient of an arbitrary scalar field λ.

$A' = A + \nabla\lambda$ then $V' = V - \frac{\partial\lambda}{\partial t}$

Electric charge conservation hold goods in every nuclear reaction and in every process of the nucleus. In generally it can be defined as charge is neither created nor destroyed rather then it can transferred from one place to another.

2. **Conservation of Energy:** It is a complicated for elementary particle because total energy can be interchange between rest energy associated with mass and kinetic as well as potential energy. The sum of these three always remains conserved in every nuclear reaction.
3. **Conservation of Linear Momentum:**

   In every nuclear reaction when no external force act o the system then total linear momentum always remains conserved. No exception of this rule has ever been observed in nature. Basically translation invariance or homogeneity of space leads to law of this conservation rule.
4. **Conservation of Angular Momentum:**

   When no external torque on the system then angular momentum always remains conserved in every nuclear reaction. The conservation of angular momentum consist orbital and spin motion together. Fermions have the half integer spin boson have the spin 0, leptons have the spin ½ photon has spin 1 and gravitons have spin 2.
5. **Conservation of Baryon Number:**

   In every nuclear reaction, total number of baryon number remains conserved or the number of baryon number minus the number of antibaryon is approximately conserved. Particles are divided into three parts-

   B=1 Particle- proton, neutron, sigma, lambda and omega

   B= -1 Particle- corresponding antiparticle have this baryon number

   B=0 Particle- K-meson, π- meson, η- meson

   B = 1/3 Particle- Quarks

B = -1/3 Particle- Antiquark

Baryon family is made up of three quarks qqq = 1/3+1/3+1/3=1(Baryon number)

Mesons are made up of quarks and antiquarks $q\bar{q}$= 1/3-1/3 = 0 (Baryon number)

Baryon number is conserved in all interactions strong, electromagnetic and weak interaction. Since the proton is the lightest baryon it can not decay in another baryons however proton have very large life-time.

Baryons anti Antibaryons

Particle		**I$_3$**	**I**	**J**	**P**	**S**	**Y**	**B**
N	P	½	½	1/2	+	0	1	1
	n	-½						
$\bar{N}$	$\bar{p}$	-½	½	1/2	-	0	-1	1
	$\bar{n}$	½						
Σ	Σ^{+}	1						
	Σ^{0}	0	1	1/2	+	-1	0	1
	Σ^{-}	-1						
≅	$\cong^{0}$	½	½	1/2	+	-2	-1	1
	$\cong^{-}$	-½						
$\bar{\Sigma}$	$\overline{\Sigma^{+}}$	1						
	$\overline{\Sigma^{0}}$	0	1	1/2	-	+1	0	-1
	$\overline{\Sigma^{-}}$	-1						
$\bar{\cong}$	$\overline{\cong^{0}}$	½	½	1/2	-	+2	+1	-1
	$\overline{\cong^{-}}$	-½						

Meson B =0

Family	Members	I_3	I	G	J	P	C_n	S=Y,B=0
π	π^+	1						
family	π^0	0	1	-	0	-	+	0
	π^-	-1						
K	K^+	½	½		0			1
family	K^0	-½				-		
	$\overline{K^0}$	½	½		0			-1
	K^-	-½						
η	η^0	0	0	+	0	-	+	0
family								
ρ	ρ^+	1						
family	ρ^0	0	1	+	1	-	-	0
	ρ^-	-1						
ω	ω	0	0	-	1	-	-	0
family								
ϕ	ϕ	0	0	-	1	-	-	0
family								

6. **Conservation of Lepton Number:** in every nuclear reaction, net lepton number always remains conserved or number of leptons minus the number of antileptons is conserved. However lepton flavour is only approximately conserved and basically it is not conserved when neutrino oscillation takes place.

   Basically lepton number is further divided in to three parts e-leptonic number, μ leptonic number and τ leptonic number and all these three leptonic number are separately conserved in every nuclear reaction.

Leptonic number for different particles

Charge	$L_e = 1$	$L_\mu = 1$	$L_\tau = 1$
Q=0	υ_e	υ_μ	υ_τ
Q= -e	e^-	μ^-	τ^-
Charge	$L_e = -1$	$L_\mu = -1$	$L_\tau = -1$
Q = 0	$\overline{\upsilon_e}$	$\overline{\upsilon_\mu}$	$\overline{\upsilon_\tau}$
Q = +e	e^+	μ^+	τ^+

Conservation of lepton has a significance for strong interaction. There are many conservation laws which are not obeyed by the weak interaction. The property which is always conserved in strong interaction is isospin other properties which are not conserved for strong EM and weak interactions but are conserved in one or two interactions are hypercharge, strangeness, parity, invariance under charge conjugation and invariance under CP conjugation.

Lepton and Antilepton J= ½

Particle	**Lepton No. L**	**Helicity**
e^-	1	$\pm\frac{1}{2}$
υ_e	1	$-\frac{1}{2}$
e^+	-1	$\pm\frac{1}{2}$
$\overline{\upsilon_e}$	-1	$+\frac{1}{2}$
μ^-	1	$\pm\frac{1}{2}$
υ_μ	1	$-\frac{1}{2}$

μ^+	-1	$\pm\frac{1}{2}$
$\overline{\upsilon_\mu}$	-1	$+\frac{1}{2}$

7. **Conservation of Isospin:**

According to this law in strong interaction the total isospin for particles is always same before and after the reaction. Isospin add according to vector rule just like the angular momentum addition. In the decay of $\pi^0 \to \gamma + \gamma$ here before the decay isospin is 1 and after the decay isospin is zero therefore this decay is not allowed by strong interaction however this decay is possible through the electromagnetic interaction.

8. **Consevationof Third Component of Isospin I_3:**

According to this law in strong and EM interaction the third component of isospin remains the same before and after particle reaction. When this law violates many times process can take place through weak interactions.

9. **Conservation of Hypercharge:**

From this conservation law strong and EM interaction the total hypercharge remains the same before and after particle reaction. When this law violates many times process can take place through weak interactions.

10. **Conservation of Parity:**

According to conservation of parity if an event is possible then its reflection in a mirror represents an equally probable event. In strong and EM interaction parity is always conserved but Yang and Lee suggested that parity is not conserved in weak interaction. Like as in the case of beta decay , decay of muons, decay of kaons**.**

11. **Conservation of charge conjugation parity:**

It is a discrete symmetry and it reverse the sign of magnetic moment, color charge and electric charge when particle converted in to antiparticle and vice- versa. In this symmetry value of weak isospin and hypercharge also changes and it obey the rule c^2=1 and so $c = \pm 1$, c remains invariant in EM interaction.

For the photon charge conjugation have the value -1 and electron and positron pair have the charge conjugation value -1^{l+s} where l represents orbital quantum number and s represents the spin quantum number.

Charge conjugation symmetry fails in weak interaction however it is applicable to strong and electromagnetic interaction. This type of symmetry obey the multiplicity property and it is highly significant for various type of interaction in elementary particle. A combination of charge conjugation parity with isospin is called G parity and this type of parity is conserved only in the strong interaction.

12. Time Reversal Symmetry T:

It is also discrete symmetry with T^2=1 and so $T = \pm 1$.

According to this symmetry an event on a particle in any system should be exactly reversible in time.

Under this symmetry operation displacement, acceleration and electric field like vector quantity remains invariant but angular momentum and magnetic field change the sign.

13. CPT Theorem:

In strong and EM interactions charge conjugation C, parity P and time reversible T operators remains invariant separately, however in weak interaction parity and charge conjugation operators does not remains conserve, but the product of CPT remains invariant irrespective of the order of any operation in all type of interaction.

Test of CPT Invariance:

According to CPT theorem consequences, electric charge and magnetic moment of particle when interchange in to its own antiparticle then electric charge and magnetic moment change the sign but particle and antiparticle have the same mass and same life time. Experimental test also confirms all about this.

Quantity	**Notation**	**P**	**C**	**T**
Position	$\vec{r}$	-1	1	1
Momentum	$\vec{p}$	-1	1	-1
Spin (axial vector)	$\vec{\sigma} = \vec{r} \times \vec{p}$	1	1	-1
Helicity	$\vec{\sigma}.\vec{p}$	-1	1	1
Electric field	$\vec{E}$	-1	-1	1
Magnetic field	$\vec{B}$	1	-1	-1
Magnetic dipole moment	$\vec{\sigma}.\vec{B}$	1	-1	1
Electric dipole moment	$\vec{\sigma}.\vec{E}$	-1	-1	-1
Transverse polarization	$\vec{\sigma}.(\vec{p_1} \times \vec{p_2})$	1	1	-1

References:

1. Henry Premarkoff, S Peter Rosan, Annual Review Nuclear Particle Science 1981.
2. David J Gross, Proceeding of the National Academic of the science of the USA 1996.
3. Brigitte Falkenberg, Ratio new series 1988.
4. David J Rowe, Journal of Physics, Conference series 403, 2012.
5. Jan B Gutowski, Symmetry and particle physics 2007.
6. D.H Perkins, Introduction to high energy physics fourth edition CUP 2000.
7. B.R Martin and G Shaw, Particle physics second edition Wiley publication 1998.
8. S Weinberg, The Quantum Theory of Fields CUP 2005.
9. Weyl, Hermann, Symmetry Princeton University Press 1952.
10. R.P Feynmann, The Character of Physical Law, Random house Inc. Newyork 1994.

11. T.D.Lee and C.N.Yang, Physical Review 1957.
12. C.S Wu Rev. Modern Physics 1950.
13. A.G.Zee, Fearful Symmetry, Macmillan Newyork 1986.
14. M.Lange, An Introduction to the Philosphy of the Physics, Oxford Black Well 2002.
15. T.P.Chang and L.F, Li, Gauge theory of elementary particle physics, Oxford 1984.
16. Arianna Borreli, Approaching Relegion 2017.
17. De Griffiths, Introduction to Elementary particles Wiley Publication 1987.
18. W. Greiner, B.Muller, Quantum Mechanics Symmetries, Springer Verlag 1989.
19. D.E.Groom et al, Europian Journal Physics 2000.
20. Mario Livio, Nature 2012.
21. D .BLicstenberg, Unitary Symmetry and elementary particles, Academic press Newyork 1979.
22. F.Iachello, Nuclear Physics A 2005.
23. M.Gell Mann, Physical Review 1962.
24. J.Barea, R.Bijker and A.Frank, Physical Review Letter 2005.
25. I Mc Donald, B Smith, International conference on principles and practise 2002.
26. Lihua Zhang and F. Xu, Advances in difference equations, 2018.
27. George Bluman, Sigma 1, 2005.
28. V.K.Mittal, R.C.Verma, S.C.Gupta, Introdution to nuclear and particle physics, PHI Learning ltd (2014).
29. Irving Keplan, Nuclear physics, Narosa publishing house 1995.
30. A.Beiser, Concepts of modern physics, Mc Graw Hill, international edition 1995.
31. B.L Cohen, Concepts of nuclear physics, Mc Graw Hill education ltd 2017.
32. D.C.Tayal, Nuclear physics, Himalaya publishing house 2017.
33. Qi Chang, Journal of nuclear and particle physics 2015.
34. K.Huang, Quarks, leptons ang gauge field word scientific publishing co. Singapur 1992.
35. G.Hooft, Nobel lecture; A confrontation with infinite, Review of modern physics 2000.